

Exercise 23

Use a linear approximation (or differentials) to estimate the given number.

$$(1.999)^4$$

Solution

Compute the derivative of $y = x^4$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^4) \\ &= 4x^3\end{aligned}$$

Consequently, the differential of $y = x^4$ is

$$dy = 4x^3 dx.$$

In order to estimate $(1.999)^4$, set $x = 2$ and $dx = -0.001$.

$$dy = 4(2)^3(-0.001) = -0.032$$

Note that dy here is the vertical distance from the function's actual value at $x = 2$ to the linear approximation's value at $x = 1.999$.

$$(1.999)^4 \approx 2^4 + (-0.032) = 15.968$$